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AUTHOR Zarghami, Ali; Benbow, Don
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ABSTRACT

Taguchi methods are methods of product quality improvement that analyze major contributions and how they can be controlled to reduce variability of poor performance. In this approach, knowledge is used to shorten testing. Taguchi methods are concerned with process improvement rather than with process measurement. This manual is designed to be used as an instructional aid in a workshop or course on Taguchi methods. It contains the following six sections: intuitive approach to the key issues of process control; introduction to the loss function; introduction to experimentation; introduction to orthogonal arrays; introduction to interactions; and a case study. An appendix contains orthogonal arrays and linear graphs and derivation of the loss function. Transparencies are provided for all key elements of the discussion, with space for participants to fill in the blanks. (KC)

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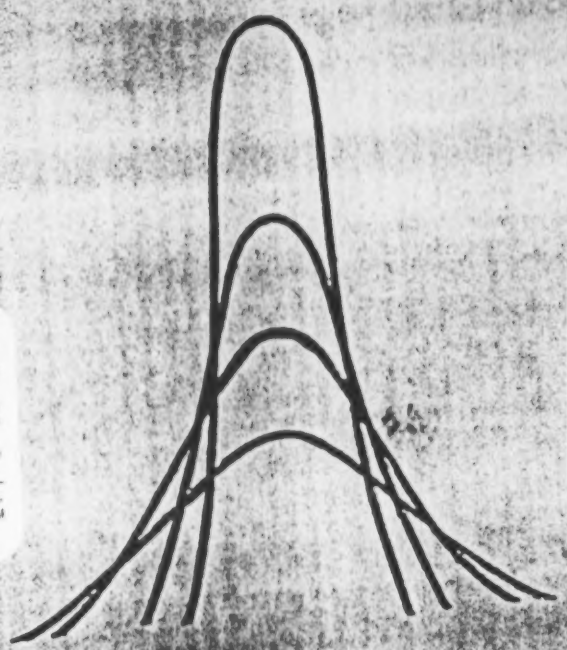

WORKBOOK FOR TAGUCHI METHODS FOR PRODUCT QUALITY IMPROVEMENT

P.O. BOX 536 MARSHALLTOWN, IA 50158 515/752-464

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COMMUNITY COLLEGES
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WORKBOOK FOR
TAGUCHI METHODS FOR
PRODUCT QUALITY IMPROVEMENT

Ali Zarghami
Fisher Controls Int'l, Inc.

Don Benbow
Marshalltown Community College

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We would like to thank Mark Steinberg,
Director of the Iowa Productivity Consortium and
Fisher Controls International
for support and assistance.

Ali Zarghami
Don Benbow

This manual is designed to be used as an instructional aid. Various items have been left blank with the intent that they will be filled in by the participant during the course presentation.

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1. An Intuitive Approach to the Key Issues of Process Control

2. Introduction to the Loss Function

3. Introduction to Experimentation

4. Introduction to Orthogonal Arrays

5. Introduction to Interactions

6. Case Study

APPENDIX
Orthogonal Arrays and Linear Graphs
Derivation of the Loss Function

"Quality is the loss imparted to society from the time a product is shipped."

-Dr. Genichi Taguchi

AN INTUITIVE APPROACH TO THE KEY ISSUES of PROCESS CONTROL

PURPOSE: To identify and examine the key concepts used in analyzing a process.

KEY ISSUES:

1. Recognition that variation exists.
2. Quantification of variation.
3. Visualization of variation.
    a) Frequency Distribution
    b) Pareto Analysis
    c) Control Charts
        i) Normality
        ii) Central Limit Theorem
4. Causes of variation.
    a) Common
    b) Special
5. Relationship between variation and design specifications.

GOAL: REDUCTION OF VARIATION.

TRANSPARENCIES:

-Data Analysis Formulas
-Control Chart Formulas
-Central Limit Theorem
-Symptoms of Instability

DATA ANALYSIS FORMULAS

	POPULATION	SAMPLE
CENTRAL TENDENCY		
MEAN:	$\mu = \sum x / N$	$\bar{x} = \sum x / n$
MEDIAN:	MIDDLE VALUE	MIDDLE VALUE
MODE:	MOST FREQUENT VALUE	MOST FREQUENT VALUE
DISPERSION OR SPREAD		
RANGE:	HIGHEST - LOWEST VALUE	
VARIANCE:	$\sigma^2 = \sum_{i=1}^{N} (x_i - \mu)^2 / N$	$s^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / (n-1)$
STANDARD DEVIATION	$\sigma = \sqrt{\sigma^2}$ $= \sqrt{\sum_{i=1}^{N} (x_i - \mu)^2 / N}$	$s = \sqrt{s^2}$ $= \sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 / (n-1)}$

CONTROL CHARTS FOR VARIABLES

Calculate the Average ($\bar{X}$) and Range (R) of each subgroup

$$\bar{X} = \frac{X_1 + X_2 + \ldots\ldots X_n}{n}$$

$$R = X_{max} - X_{min}$$

Calculate the Average Range ($\bar{R}$) and the Process Average ($\bar{\bar{X}}$)

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \ldots\ldots + \bar{X}_k}{k}$$

$$\bar{R} = \frac{R_1 + R_2 + \ldots\ldots + R_k}{k}$$

Calculate the Control Limits

$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$ $\quad UCL_R = D_4\bar{R}$

$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$ $\quad LCL_R = D_3\bar{R}$

Factors

n	3	4	5
D_4	2.57	2.28	2.11
D_3	0	0	0
A_2	1.02	.73	.58
d_2	1.693	2.059	2.326

CONTROL CHARTS FOR ATTRIBUTES

The p Chart

$$p = \frac{\text{number of rejects in subgroup}}{\text{number inspected in subgroup}}$$

$$\bar{p} = \frac{\text{total number of rejects}}{\text{total number inspected}}$$

$$UCL_p = \bar{p} + \frac{3\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$$

$$LCL_p = \bar{p} - \frac{3\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$$

The np Chart

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

The c Chart

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

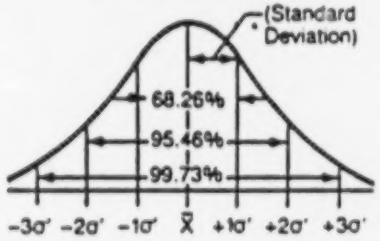
The u Chart

$$\bar{u} = \frac{\text{total nonconformities}}{\text{total units inspected}}$$

$$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{n}}$$

$$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{n}}$$

NORMAL CURVE



PROCESS CAPABILITY

Cp Index

$$Cp = \frac{\text{spec. width}}{\text{natural tolerance}}$$

$$Cp = \frac{\text{upper spec.} - \text{lower spec.}}{6\hat{\sigma}'}$$

Cpk Index

$\hat{\sigma}' = \bar{R}/d_2$

$$Z_U = \frac{USL - \bar{\bar{X}}}{\hat{\sigma}'} \qquad Z_L = \frac{\bar{\bar{X}} - LSL}{\hat{\sigma}'}$$

Z_{min} = Minimum of Z_U or Z_L

$$C_{pk} = \frac{Z_{min}}{3} = \text{Minimum of } \frac{USL - \bar{\bar{X}}}{3\hat{\sigma}'} \; \frac{\bar{\bar{X}} - LSL}{3\hat{\sigma}'}$$

Proportions out of specification

Pz_U =
Pz_L = ⟨Values from Tables⟩

CENTRAL LIMIT THEOREM

The distribution of sample means is approximately normal even though the parent population is not normal.
The approximation improves as the sample size increases.

SIGNS OF INSTABILITY

1. A point outside control limits.
2. Seven consecutive points above or below average.
3. Unexplained trends. (Seven up or down.)
4. Middle third of chart contains more than 90% of points when 25 points have been plotted.
5. Middle third of chart contains fewer than 40% of points when 25 points have been plotted.
6. Unexplained patterns.

INTRODUCTION TO THE LOSS FUNCTION

PURPOSE: To show how Dr. Taguchi translates product variation into dollar loss.

Conventionally, quality costs have been measured based on conformance to specifications, using scrap and re-work costs. Therefore, tolerances become go/no-go limits, where any part meeting specifications is as good as any other. Dr. Taguchi demonstrates that parts near the nominal dimension have more value than others.

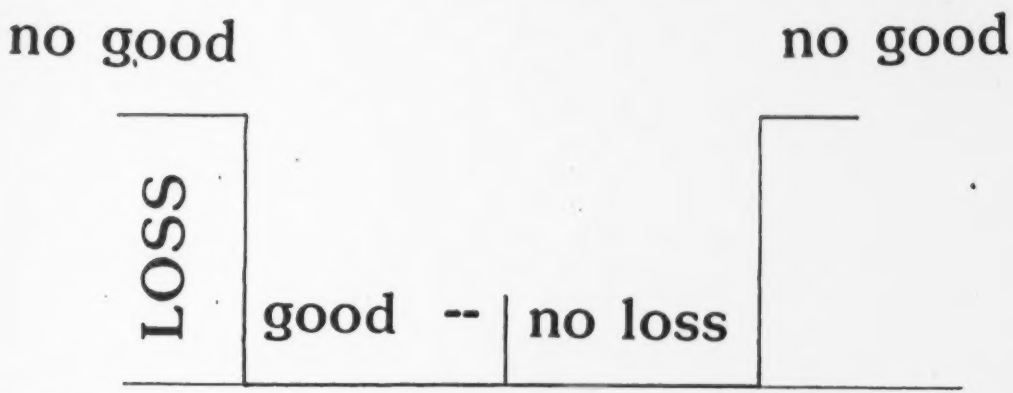
TRANSPARENCIES:

Conventional interpretation of loss vs Taguchi's

Color density distributions

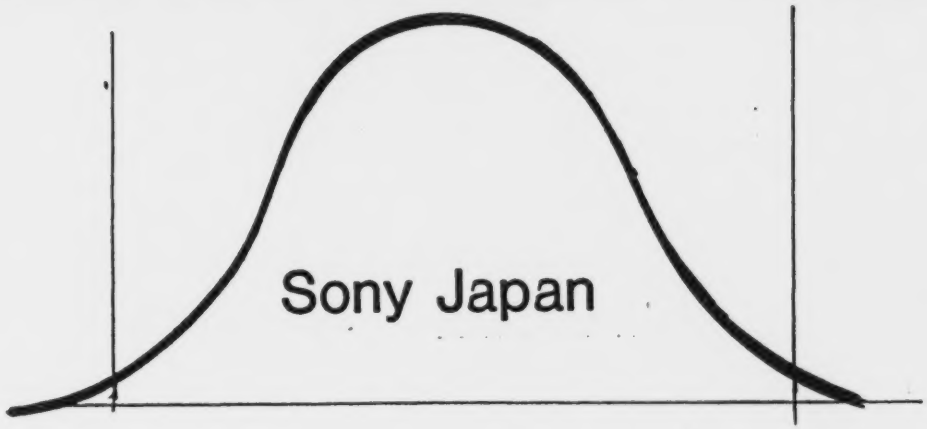
Loss calculations

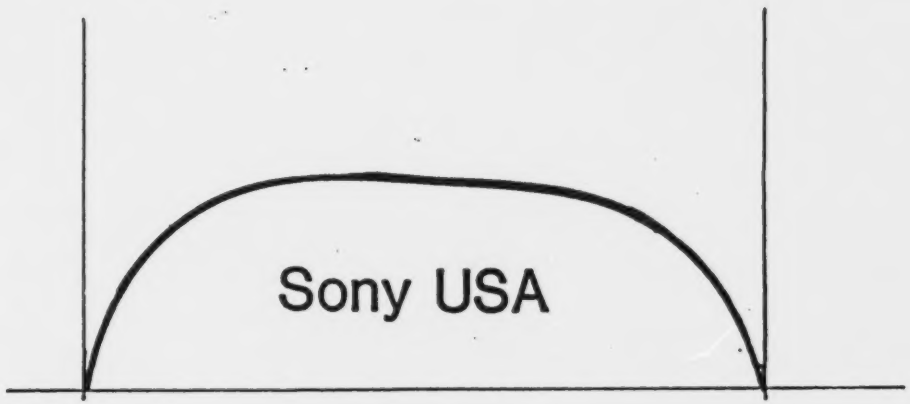
Quality/cost curves



CONVENTIONAL INTERPRETATION OF LOSS

TAGUCHI'S "continuous" INTERPRETATION





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COLOR DENSITY DISTRIBUTIONS

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LOSS CALCULATIONS

Taguchi estimates unit loss due to variation as

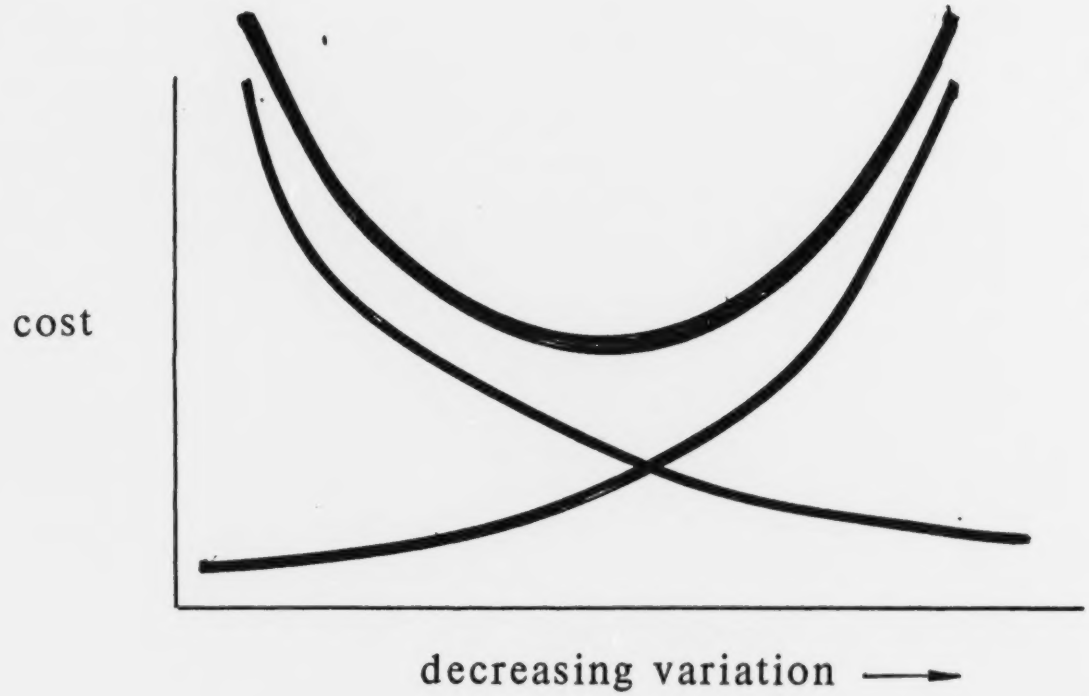
$$L(x) = A\sigma^2/d^2$$

where d = tolerance/2
A = scrap cost
σ std. deviation

PROBLEM:

- A part with specs 1.455 - 1.465 which costs $6 to scrap is made on a process with σ=.0015. A proposal for process improvement will reduce to .001 but will cost $8000 per year. Is this expenditure justified if annual production is 100,000 units?

- According to a Japanese newspaper the Sony-USA color density had σ=2.9 and Sony-Japan had σ=1.7. If specs were ±5 and an out of spec tube costs $4, how much more valuable are Japanese tubes?



QUALITY/COST CURVES

INTRODUCTION TO EXPERIMENTATION

PURPOSE: To show how a series of tests can be designed to reduce process variation.

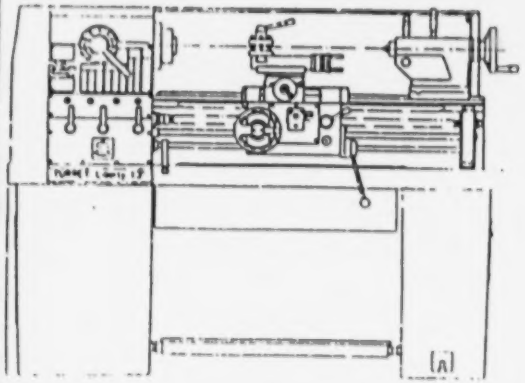
Experimentation is the systematic approach to determining how changes in the system actually relate to the final function of the product or process. In this section, basic concepts behind full factorial design will be discussed.

The application of experimental design will be discussed through a couple of actual shop problems.

TRANSPARENCIES:

Practical experimentation concepts
Calculation of main effects
Graphing of Main effects
Problem
Graphs of the problem results
Effect of non-linearity on system variation

PRACTICAL EXPERIMENTATION CONCEPTS



FACTORS: Feed – .01 .04
Speed – 1300 1800
Clnt temp – 100 140

$$\#\text{combinations} = (\#\text{levels})^{(\#\text{factors})}$$

Experiment Number	Factors		
	FEED	SPEED	CTEMP

CALCULATION of MAIN EFFECTS

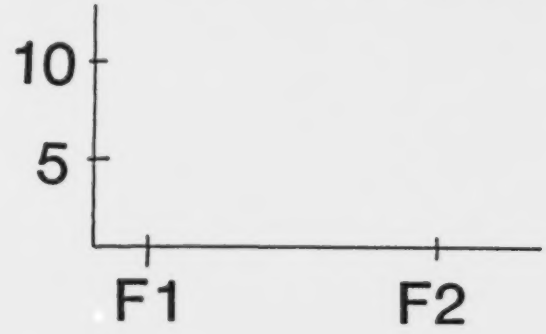
Experiment Number	Factors			RESULTS
	Feed	Speed	Ctemp	
1	.01	1300	100	10
2	.01	1300	140	4
3	.01	1800	100	6
4	.01	1800	140	2
5	.04	1300	100	7
6	.04	1300	140	6
7	.04	1800	100	6
8	.04	1800	140	3

#	A	B	C
1	1	1	1
2	1	1	2
3	1	2	1
4	1	2	2
5	2	1	1
6	2	1	2
7	2	2	1
8	2	2	2

GRAPHING MAIN EFFECTS

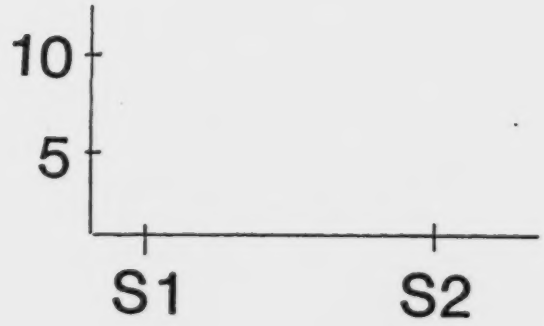
F1 = 5.5

F2 = 5.5



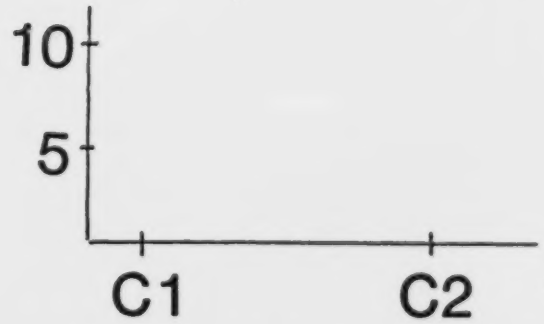
S1 = 6.75

S2 = 4.25



C1 = 7.25

C2 = 3.75



Interpretation

PROBLEM

A chemical process is not yielding the expected production. People closest to the process have identified PRESSURE, TEMP, and TIME as the influential factors. They agree that the experimental levels should be 80 - 120 psi 200°-300° F and 14 - 18 min.

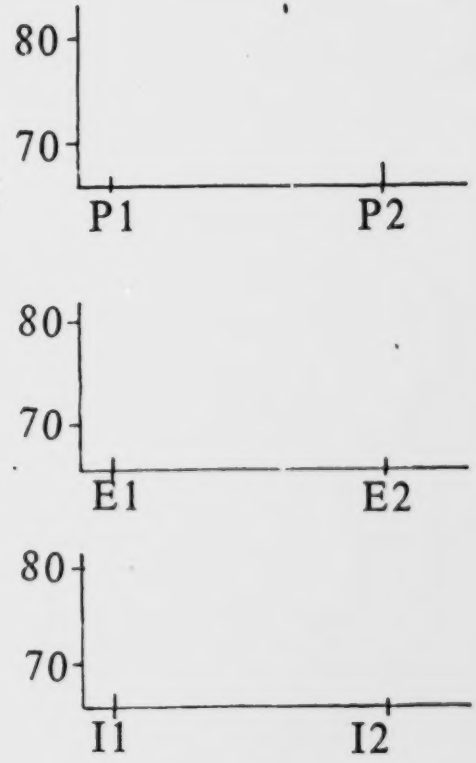
1. Set up an experimental design.

#	

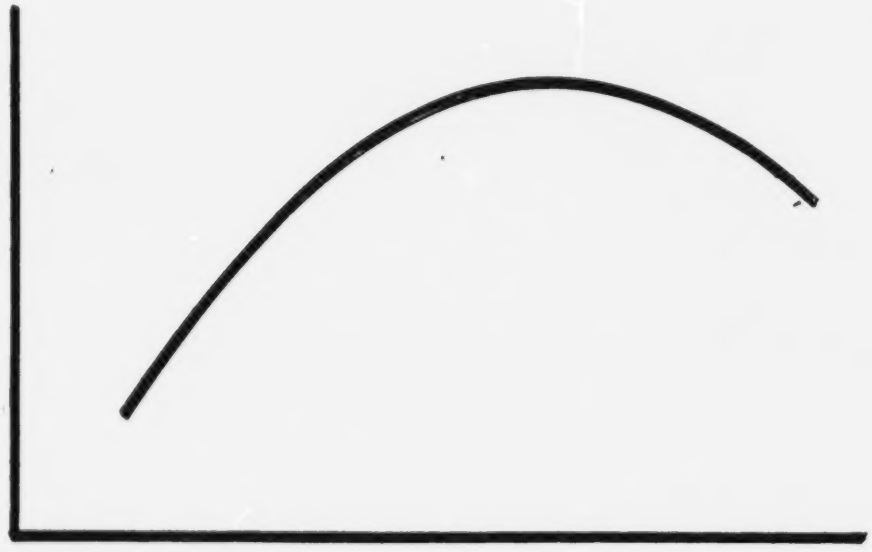
2. Calculate main effects given these results:

   80, 70 76, 84, 68, 64, 69, 82.

GRAPHS OF THE PROBLEM RESULTS



EFFECT OF NON-LINEARITY



INTRODUCTION TO ORTHOGONAL ARRAYS

PURPOSE: To reduce the number of experimental tests to a more practical size for industry.

Taguchi methods of experimentation focus on a pareto of major contributors and how they can be controlled to reduce variability or poor performance. He is not interested in a detailed, precise analysis of less significant factors.

Taguchi down-plays interaction since major factors dominate in electrical/mechanical assemblies, unlike chemical and medical applications where many unpredictable interactions occur between variables.

Taguchi furthered the application of orthogonal arrays for mechanical and electrical assemblies by limiting the number of experiments to a practical quantity.

Orthogonal arrays are a specialized form of fractional factorial where many manipulations, pooled values , tricks, combinations and other shortcuts are used to reduce testing and analysis.

Taguchi also recommends verification experiments to confirm initial experiments and to verify robustness of the parameter/ tolerance design actions.

Taguchi attributes much of the success at Hatachi, NEC and Nippodenso to extensive use of orthogonal arrays in design of products and processes. At Nippondenso, some 2,700 different orthogonal array experiments were performed in 1976. In 1985, over one million experiments were done in Japan, some with 2,000 variables. Every Japanese engineer is now trained in Taguchi methods.

TRANSPARENCIES:

Franctional factorials - Orthogonal Arrays

L_4 and L_9

Problem

Phases of the Design Process

FRACTIONAL FACTORIALS – ORTHOGONAL ARRAYS

Do we need to test every combination of factor levels?

	a	b	c
1	1	1	1
2	1	1	2
3	1	2	1
4	2	1	1

	a	b	c
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

L_4 and L_9

$L_4(2)^3$

	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

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$L_9(3)^4$

	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

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PROBLEM

An engineering and manufacturing design team has determined four factors that may influence the voltage level of a circuit.

i) Select an appropriate orthogonal array.

ii) Using the results below, calculate the main effects, plot the response curves, select the best parameter levels and identify the control factors. Assume a target value of 0.

RESULTS: -4, 6, 2, -6, 11, -6, -4, 1, -3

THREE PHASES OF THE DESIGN PROCESS

SYSTEM DESIGN: BASED ON KNOWLEDGE, RESEARCH, EXPERIENCE AND LATEST TECHNOLOGIES. RELIES ON KNOWLEDGE FROM RELEVENT FIELDS TO SELECT THE PROPER ALTERNATIVES.

PARAMETER DESIGN: PARAMETER DESIGN PROVIDES A TECHNIQUE TO DESIGN A PRODUCT OR PROCESS BY SELECTING THE OPTIMUM CONDITIONS OF PARAMETER LEVELS SO THAT THE PRODUCT IS LEAST SENSITIVE TO NOISE VARIABLES. PARAMETER DESIGN TAKES ADVANTAGE OF NON-LINEAR EFFECTS IN ORDER TO OBTAIN "ROBUSTNESS" OR INSENSITIVITY TO NOISE.

TOLERANCE DESIGN: AIMS TO DEFINE THE TOLERANCE/ALLOWANCE ASSOCIATED WITH THE FACTORS WHOSE NOMINAL VALUES WERE CHOSEN IN PARAMETER DESIGN.

INTRODUCTION TO INTERACTIONS

PURPOSE: To define and show how orthogonal arrays can be used to analyze interactions.

An interaction occurs between two factors if the response curve of one factor depends on the level of the other factor.

TRANSPARENCIES:

Graphical interpretation of interactions

$L_4(2)^3$

$L_8(2)^7$ with linear graph #1

$L_8(2)^7$ with linear graph #2

Problem

GRAPHICAL INTERPRETATION OF INTERACTIONS

A chemical processor is trying to reduce the NO_x emmision from the stacks in two of his plants. A full factorial experiment is set up for each plant to test for two levels of the two factors

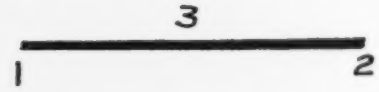
A= Acidity level (pH)
B= Bromine Concentration (%)

The experimental design and results are shown below:

	PLANT #1			PLANT #2		
	A	B	RESULT	A	B	RESULT
1	1	1	10	1	1	10
2	1	2	20	1	2	20
3	2	1	30	2	1	18
4	2	2	40	2	2	15

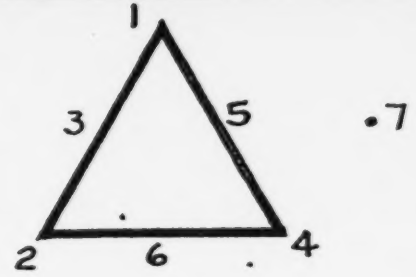
Graph the main effects:

$L_4 (2)^3$



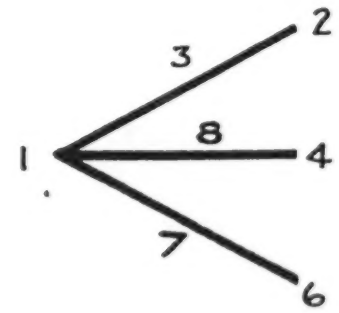
#	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

$L_8\ (2)^7$



#	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

$L_8 (2)^7$



#	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

In an attempt to reduce the porosity in a die casting, the people involved have identified these factors as being influential:

-die cooldown time
-long shot velocity
-short shot velocity
-ram pressure
-pot temperature

They believe there may be interactions between each of the velocities and the ram pressure. They decide to test each factor at two levels.

a) Select an appropriate orthogonal array

b) Label its columns appropriately

c) Using these porosity scores (target is 0), calculate the main effects, the interaction effects and plot the response curves.

RESULTS: 29, 79, 117, 7, 47, 45, 145, 87

SUMMARY

Statisticians focus on post mortum data analysis. What is needed is to focus on generation of meaningful data and prevention.

Taguchi methods don't focus on statistical significance, but a pareto of major contributions and how they can be controlled to reduce variability of poor performance. He is not interested in a detailed, precise analysis of less significant factors.

Statisticians create a model and test to see if model is true. Statisticians are concerned with sampling error focus on confidence limits and test for significance of all variables. Taguchi addresses the largest factors we know of, regardless of statistical significance.

Statisticians assume we know nothing and that common causes are random. Taguchi says we do have knowledge, so let's use it to shorten testing. We are smarter than to wait until problems occur; we can generate tests to cause conditions to happen.

Taguchi is not as concerned with process measurement as with process improvement.

Taguchi says processes are always changing. The most important aspect is not how much the process is changing, but why and how do we adjust it or stabilize it.

Control charts have been likened to an observer of a process and generate a signal when a significant event occurs. This is characterized as passive surveillance. Taguchi methods are likened to active surveillance where the probability of an event occurring is increased through knowledge of product or process and experience.

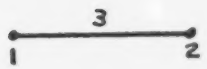
APPENDIX

Orthogonal Arrays and Linear Graphs

Derivation of the Loss Function

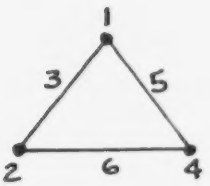
$L_4(2)^3$

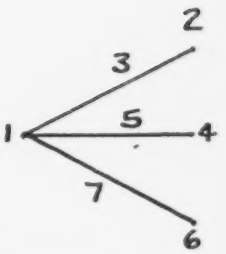
Experiment #	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1



$L_8(2)^7$

Experiment #	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2





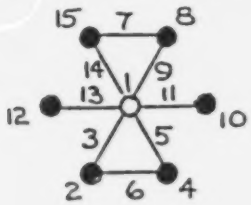
$L_{12}(2)11$

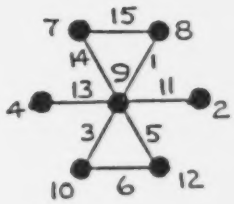
Experiment #	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1

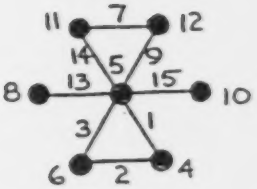
$L_{16}(2)^{15}$

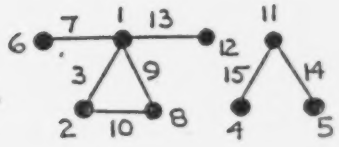
Experiment #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

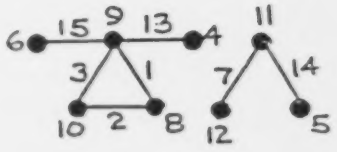
LINEAR GRAPHS FOR $L_{16}(2)^{15}$:

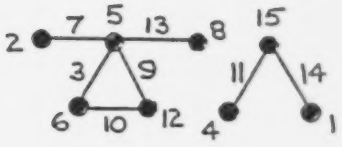


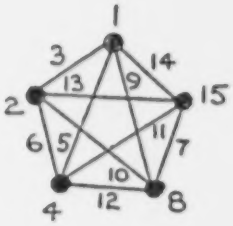


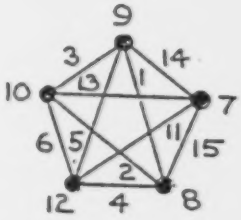


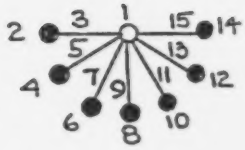


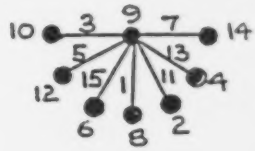


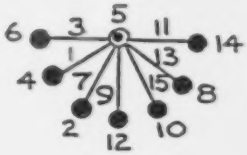


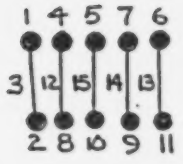


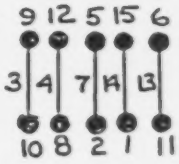


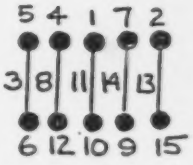


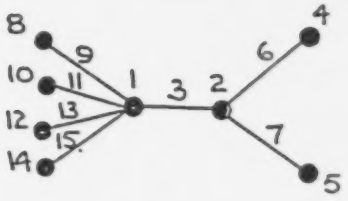


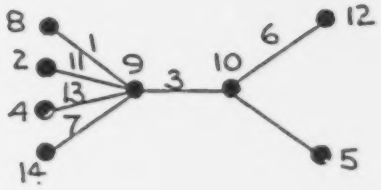


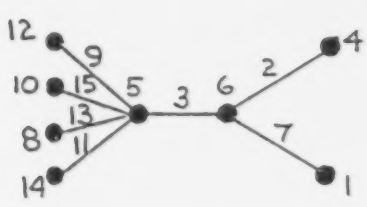


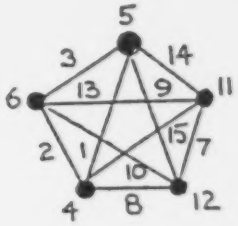








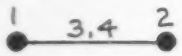




$L_9(3)^4$

Experiment #	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Linear Graph



$L_{18}(2^1 \times 3^7)$

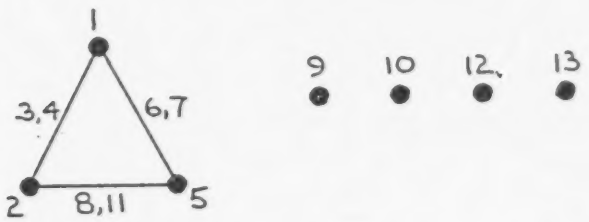
Experiment #	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1

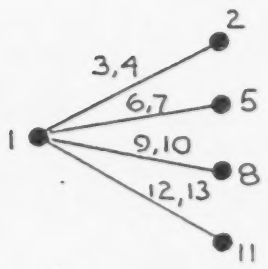
Linear Graph

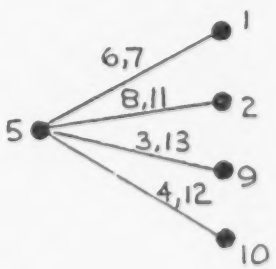
$L_{27}(3)^{13}$

Experiment #	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1
10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2

Linear Graphs







$L_{36}(3)^{13}$

Experiment #	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	1
3	3	3	3	3	3	3	3	3	3	3	3	3	1
4	1	1	1	1	2	2	2	2	3	3	3	3	1
5	2	2	2	2	3	3	3	3	1	1	1	1	1
6	3	3	3	3	1	1	1	1	2	2	2	2	1
7	1	1	2	3	1	2	3	3	1	2	2	3	1
8	2	2	3	1	2	3	1	1	2	3	3	1	1
9	3	3	1	2	3	1	2	2	3	1	1	2	1
10	1	1	3	2	1	3	2	3	2	1	3	2	1
11	2	2	1	3	2	1	3	1	3	2	1	3	1
12	3	3	2	1	3	2	1	2	1	3	2	1	1
13	1	2	3	1	3	2	1	3	3	2	1	2	2
14	2	3	1	2	1	3	2	1	1	3	2	3	2
15	3	1	2	3	2	1	3	2	2	1	3	1	2
16	1	2	3	2	1	1	3	2	3	3	2	1	2
17	2	3	1	3	2	2	1	3	1	1	3	2	2
18	3	1	2	1	3	3	2	1	2	2	1	3	2
19	1	2	1	3	3	3	1	2	2	1	2	3	2
20	2	3	2	1	1	1	2	3	3	2	3	1	2
21	3	1	3	2	2	2	3	1	1	3	1	2	2
22	1	2	2	3	3	1	2	1	1	3	3	2	2
23	2	3	3	1	1	2	3	2	2	1	1	3	2
24	3	1	1	2	2	3	1	3	3	2	2	1	2
25	1	3	2	1	2	3	3	1	3	1	2	2	3
26	2	1	3	2	3	1	1	2	1	2	3	3	3
27	3	2	1	3	1	2	2	3	2	3	1	1	3
28	1	3	2	2	2	1	1	3	2	3	1	3	3
29	2	1	3	3	3	2	2	1	3	1	2	1	3
30	3	2	1	1	1	3	3	2	1	2	3	2	3
31	1	3	3	3	2	3	2	2	1	2	1	1	3
32	2	1	1	1	3	1	3	3	2	3	2	2	3
33	3	2	2	2	1	2	1	1	3	1	3	3	3
34	1	3	1	2	3	2	3	1	2	2	3	1	3
35	2	1	2	3	1	3	1	2	3	3	1	2	3
36	3	2	3	1	2	1	2	3	1	1	2	3	3

$L_{32}(2^1 \times 4^9)$

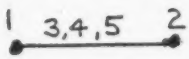
Experiment #	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3	3	3
4	1	1	4	4	4	4	4	4	4	4
5	1	2	1	1	2	2	3	3	4	4
6	1	2	2	2	1	1	4	4	3	3
7	1	2	3	3	4	4	1	1	2	2
8	1	2	4	4	3	3	2	2	1	1
9	1	3	1	2	3	4	1	2	3	4
10	1	3	2	1	4	3	2	1	4	3
11	1	3	3	4	1	2	3	4	1	2
12	1	3	4	3	2	1	4	3	2	1
13	1	4	1	2	4	3	3	4	2	1
14	1	4	2	1	3	4	4	3	1	2
15	1	4	3	4	2	1	1	2	4	3
16	1	4	4	3	1	2	2	1	3	4
17	2	1	1	4	1	4	2	3	2	3
18	2	1	2	3	2	3	1	4	1	4
19	2	1	3	2	3	2	4	1	4	1
20	2	1	4	1	4	1	3	2	3	2
21	2	2	1	4	2	3	4	1	3	2
22	2	2	2	3	1	4	3	2	4	1
23	2	2	3	2	4	1	2	3	1	4
24	2	2	4	1	3	2	1	4	2	3
25	2	3	1	3	3	1	2	4	4	2
26	2	3	2	4	4	2	1	3	3	1
27	2	3	3	1	1	3	4	2	2	4
28	2	3	4	2	2	4	3	1	1	3
29	2	4	1	3	4	2	4	2	1	3
30	2	4	2	4	3	1	3	1	2	4
31	2	4	3	1	2	4	2	4	3	1
32	2	4	4	2	1	3	1	3	4	2

Linear Graphs

$L_{16}(4)^5$

Experiment #	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	3	3	3	3
4	1	4	4	4	4
5	2	1	2	3	4
6	2	2	1	4	3
7	2	3	4	1	2
8	2	4	3	2	1
9	3	1	3	4	2
10	3	2	4	3	1
11	3	3	1	2	4
12	3	4	2	1	3
13	4	1	4	2	3
14	4	2	3	1	4
15	4	3	2	4	1
16	4	4	1	3	2

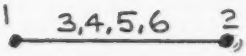
Linear Graph



$L_{25}(5)^6$

Experiment #	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	3	3	3	3	3
4	1	4	4	4	4	4
5	1	5	5	5	5	5
6	2	1	2	3	4	5
7	2	2	3	4	5	1
8	2	3	4	5	1	2
9	2	4	5	1	2	3
10	2	5	1	2	3	4
11	3	1	3	5	2	4
12	3	2	4	1	3	5
13	3	3	5	2	4	1
14	3	4	1	3	5	2
15	3	5	2	4	1	3
16	4	1	4	2	5	3
17	4	2	5	3	1	4
18	4	3	1	4	2	5
19	4	4	2	5	3	1
20	4	5	3	1	4	2
21	5	1	5	4	3	2
22	5	2	1	5	4	3
23	5	3	2	1	5	4
24	5	4	3	2	1	5
25	5	5	4	3	2	1

Linear Graphs



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DERIVATION OF TAGUCHI'S LOSS FUNCTION

GOAL: To develop a formula for the dollar loss due to variation from the target.

Let x= variation from the target in the appropriate units.
(so target is at x=0)

f(x) is in dollars

A= dollar loss when a part exceeds spec. limit s
(for bilateral tolerances, s= ½ tolerance)

Assumptions:

a) Loss is zero when x=0 because the characteristic is on target

b) Loss is at a minimum point when x=0

c) The loss function may be represented by a Maclaurin expansion

d) The function contains the point (s, A)

by c) $f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f''''(0)x^4}{4!} + \ldots$

$f(0)=0$ by a) and $f'(0) = 0$ by b)

omitting 3rd and higher degree terms

$f(x) \doteq kx^2$ where $k= \frac{f''(0)}{2}$

This is a parabola. To evaluate k, substitute point (s, A) because of d)

$$f(x) \doteq \frac{A}{s^2} x^2$$

This function represents the loss due to variation for a particular unit whose variation from the target is x. For a number of units, the loss would be

$$f(x) \doteq \frac{A}{s^2} \overline{x^2}$$

and since $\overline{x^2}$ is the average of the sum-of-squares of the deviations from the mean value, it can be represented by σ^2.

therefore $f(x) = \frac{A}{s^2} \sigma^2$